Title: Lines, Lines, Everywhere!!

A discovery/exploration lesson investigating equations of the form $y = mx + b$ to see how the values of b and m affects the graph.

Link to Outcomes:

- **Communication/ Cooperation** — Students will work in groups of two to investigate mathematical concepts. They will share their investigations and conclusions verbally within their groups and in writing on their activity worksheets.
- **Reasoning/ Analysis** — Students will analyze linear equations and their graphs to determine their algebraic and graphic properties.
- **Connections** — Students will observe/make connections between the algebraic concept of rate of change (slope) and a similar real-world situation.
- **Measurement/ Graphing** — Students will use various media (computer, graphing calculator, paper and pencil) to graph/sketch linear equations.
- **Algebra** — Students will write equations in slope-intercept form and demonstrate their understanding of the role of slope and y-intercept in the equations and graphs.
- **Technology** — Students will use a desktop computer (MET and/or Derive) and/or a graphing calculator to assist in the drawing and analysis of linear equations.

Brief Overview:

In this lesson, the students will explore the effects of changing values for the y-intercept and for the slope of a line. They will explain/discover the concepts of slope, intercepts, parallel, and perpendicular lines.

Grade/Level:

Grades 9-10: Pre-Algebra, Basic Algebra and Algebra 1

Duration:

3 one-hour sessions

Prerequisite Knowledge:

Introduction to linear equations. Knowledge of Cartesian coordinates. Ability to plot points on graphs. Introductory skills in MET, Derive, TI-81, or TI-82.

Objectives:

Students will be able to:

- use computer/graphing calculator to graph linear equations given in slope-intercept form.
- describe the effect on the graph of a linear equation by changing the value for the *y*-intercept.
- describe the effect on the graph of a linear equation by changing the value for the slope.
- determine the relationships of parallel and perpendicular between two lines based on knowledge of the respective slopes.
- write in slope-intercept form the equation of a line that is parallel to a given line.
- write in slope-intercept form the equation of a line that is perpendicular to a given line.

Materials/Resources/Printed Materials:

- Computer and software (MET, Derive)
- Graphing calculator (TI-81 or TI-82)
- Activity worksheet

Development/Procedures:

The teacher will pair students into cooperative workgroups. In each group, students will use a worksheet that consists of a series of activities that allows them to explore the effect of making changes in the slope and *y*-intercept of a linear equation. The activities also require student teams to use a computer/graphing calculator to plot linear equations; students will then make sketches of the plots. These activities will task students to explore the algebraic and graphic concepts of parallel and perpendicular lines.

Warmup/Lead-in Activity:

McArthur Shopping Mall has just opened. You have a retail job which pays $150.00 plus a 10% commission of your sales per week. In your first week you sold $700 of merchandise. What is your pay? In the following weeks, you sold $300 and $1500 worth of merchandise. What is your pay for each of these weeks? Plot these salaries as points on a linear graph.

Activity 1:

a. The teacher will give students a "starting" equation then choose several different values to substitute for the *y*-intercept. Students will graph the new equation and describe the resulting graph.

b. The teacher will give students a new "starting" equation that is the same as the first except that the slope is negative. The students repeat their actions from step a. above.

Activity 2:

The teacher will give students a "starting" equation then choose several different values to substitute for the slope. Students will graph the new equation and describe the resulting graph.

Activity 3:

a. The teacher will give the students the equations of three pairs of perpendicular lines. Students will graph each pair of lines and discuss the relationship between each pair of lines.

b. Students are tasked to write and/or identify equations which form perpendicular lines.

Evaluation:

Students will be evaluated on the quality of their discussions and completion of the activity worksheets. The teacher will monitor student progress and involvement by circulating among the teams. Students will be evaluated as a group; student interaction and cooperation within their groups is essential to their development as student mathematicians.

Extension/Follow-up:

Students are given the opportunity to continue their exploration of linear equations using a real-world application.

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LINES, LINES, EVERYWHERE WORKSHEET

Activity 1

In this activity, we will study equations of the form $y = mx + b$ to see how the value of b affects the graph.

Use the table below to complete the following tasks.

a. Select four different values for b.
b. Write the linear equation for each value of b.
c. Using the computer/graphing calculator, graph each line.
d. Determine the y-intercept of each line.
e. Analyze each graph to determine if it has moved vertically up or down in reference to the starting equation.
f. Sketch each computer generated graph on the attached grid.

EQUATION	VALUE OF b	Y-INTERCEPT	MOVES UP/DOWN
$y = x + 2$	2	2	----------

The slope in your starting equation has been changed from +1 to -1. Repeat steps a - f from above.

EQUATION	VALUE OF b	Y-INTERCEPT	MOVES UP/DOWN
$y = -x + 2$	2	2	----------

Answer the following questions from the tables, graphs, and sketches that you have developed.

1. How does changing the value of b affect the graphs?

2. How does changing the slope from + or - have any affect on the y-intercepts?

3. Does changing the value of b affect the y-intercept? If so, how?

4. Does changing the value of b affect the x-intercept? If so, how?

5. Does changing the value of b affect the slope of the graphs? If so, how?

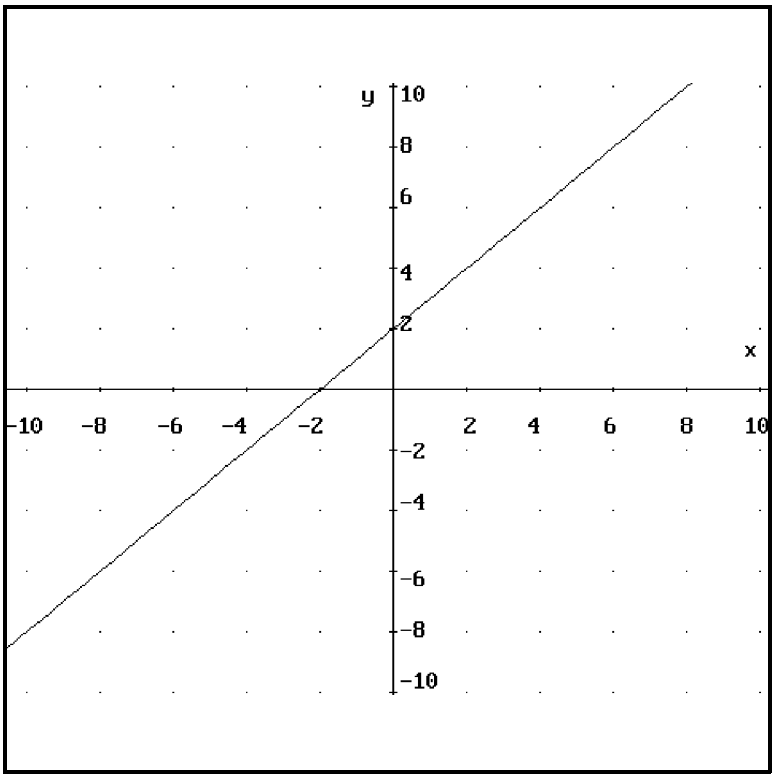
6. Lines that have the same slope are _____.

7. If b is equal to zero, what happens to the graph? Describe the resultant line?

8. If b is greater than 0 ($b > 0$), what happens to the graph?

9. If b is less than zero ($b < 0$), what happens to the graph?

10. How does the value of b relate to the graph of $y = mx + b$? Discuss.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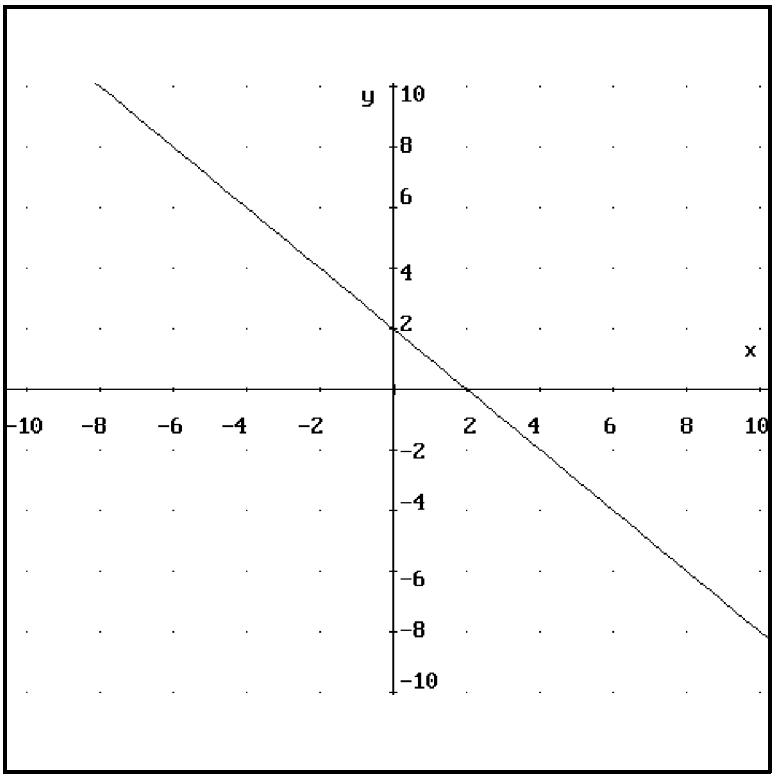
y = x + 2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y = -x + 2

LINES, LINES, EVERYWHERE WORKSHEET

Activity 2

In this activity, we will study equations of the form $y = mx + b$ to see how changing the value of m affects the graph.

Use the table below to complete the following tasks.

a. Select six different values for m.
b. Write the linear equation for each value of m.
c. Using the computer/graphing calculator, graph each line.
d. Analyze each graph to determine the changes in reference to the starting equation.
e. Sketch each computer generated graph on the attached grid.

EQUATION	VALUE OF b	Y-INTERCEPT	VALUE OF m	COMPARE TO $y = 2x - 4$
$y = 2x - 4$	-4	-4	2	---------
	-4	-4		
	-4	-4		
	-4	-4		
	-4	-4		
	-4	-4		
	-4	-4		

Answer the following questions:

1. How does changing the value of m affect the graphs?

2. Does changing the value of m affect the y-intercept? If so, how?

3. Does changing the value of m affect the x-intercept? If so, how?

4. If $m = 0$, what happens to the graph?

5. If $m > 0$, what happens to the graph?

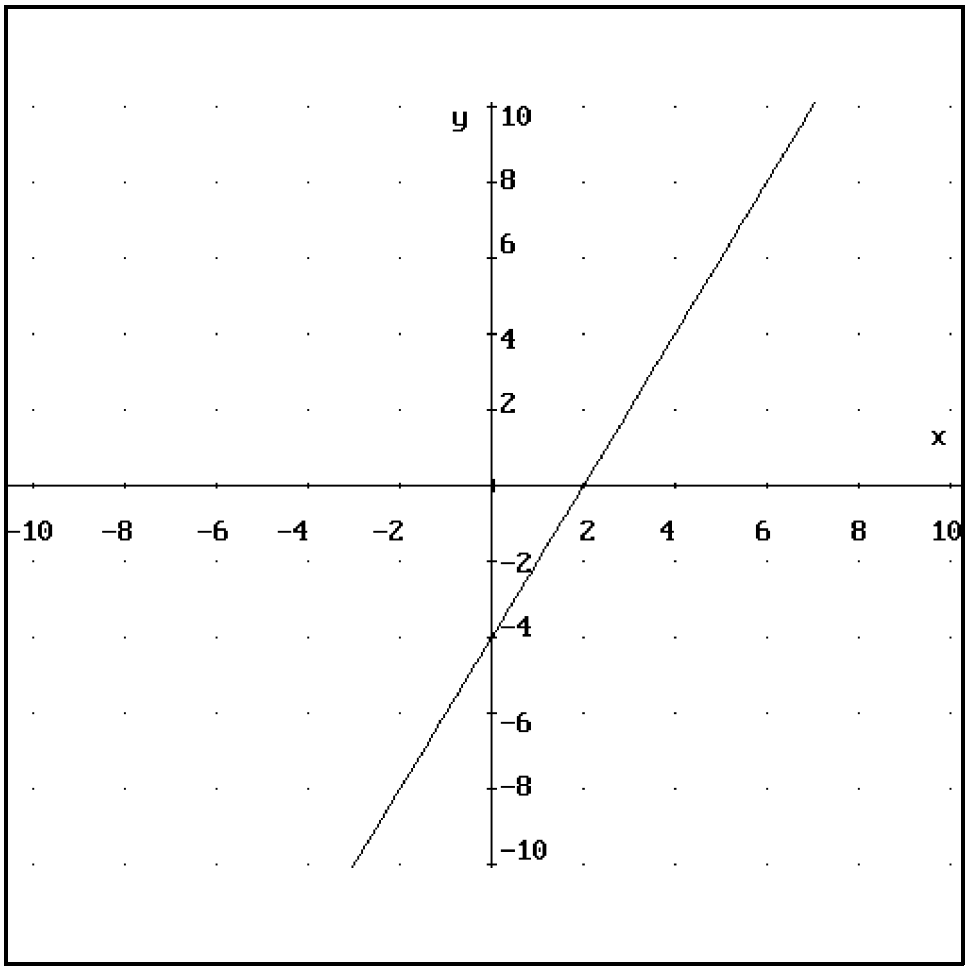
6. If $m < 0$, what happens to the graph?

7. What is m? Explain its significance?

Extension:

Discuss what happens to m as a line gets closer and closer to being vertical.

What happens to the line when it is vertical?



y = 2x - 4

LINES, LINES, EVERYWHERE WORKSHEET

Activity 3

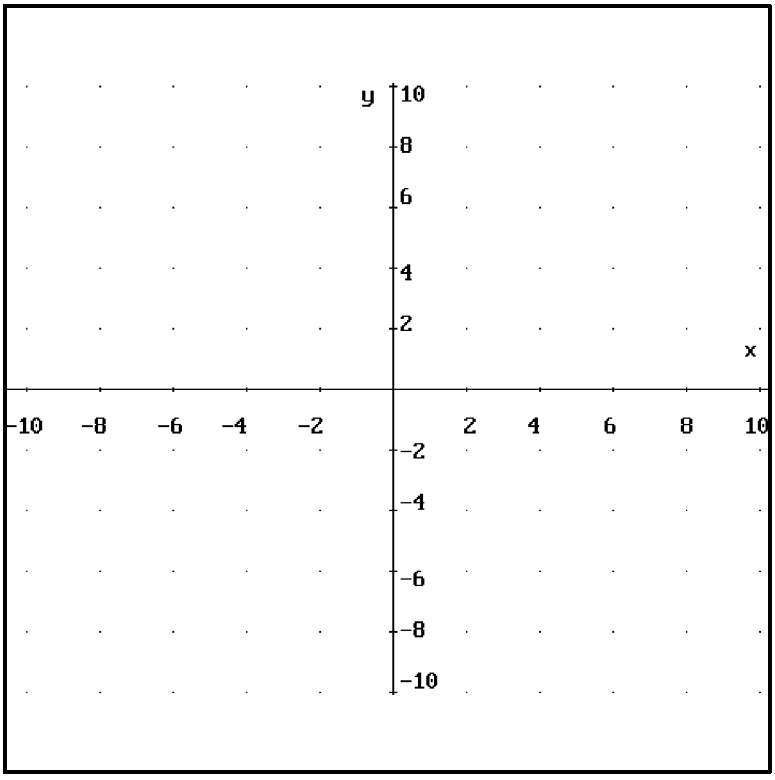
This activity will explore two special lines.

Use your calculator or computer to graph the following equations.

$$y = \frac{2}{3}x + 4$$

$$y = -\frac{3}{2}x + 4$$

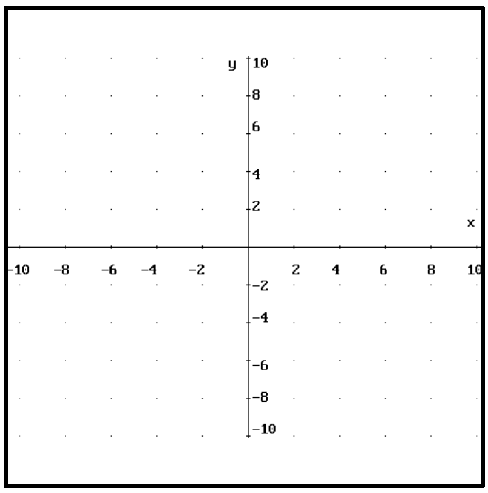
1. Sketch the graph of the two lines.



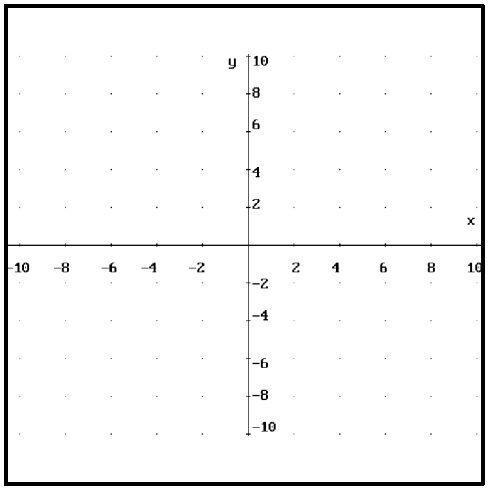
2. Study the two equations and state the changes that were made to form the second equation?

Graph each set of lines below and sketch.

3. $y = \frac{1}{2}x - 3$
   $y = -2x - 3$

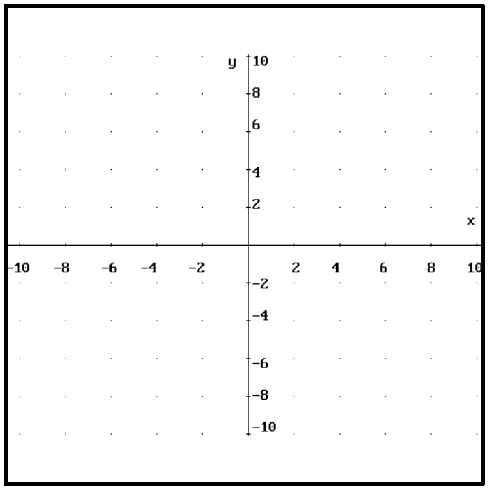


4. $y = 4x + 6$
   $y = -\frac{1}{4}x + 6$



5. $y = \frac{1}{2}x - 3$

   $y = -\frac{4}{3}x + 7$



6. What changes were made in each set of equations?

7. In each case, the graphs form what type of lines?

8. What must occur in the equations of two lines for their graphs to be perpendicular?

9. Identify whether each set of lines is perpendicular by circling yes or no.

A. $y=4x+3$

$y=-\frac{1}{4}x+3$ Yes/No

B. $y=-\frac{2}{7}x+5$

$y=\frac{7}{2}x+5$ Yes/No

C. $y=-\frac{2}{3}x-4$

$y=-\frac{3}{2}x+4$ Yes/No

D. $y=5x$

$y=-\frac{1}{5}x$ Yes/No

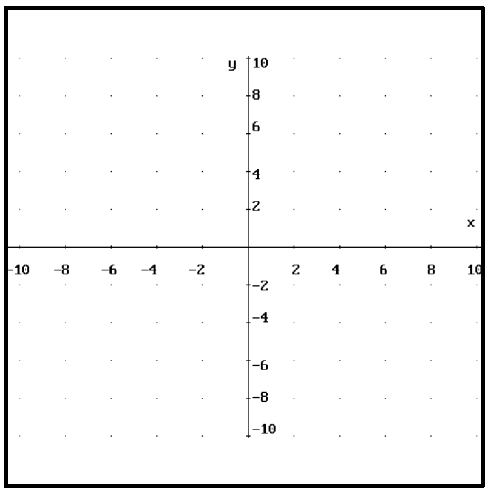
E. $y=4-5x$

$y=\frac{1}{5}x+4$ Yes/No

10. Write an equation that is perpendicular to $y=-\frac{1}{8}x+4$

11. Write 3 sets of perpendicular lines and graph each set.

A.



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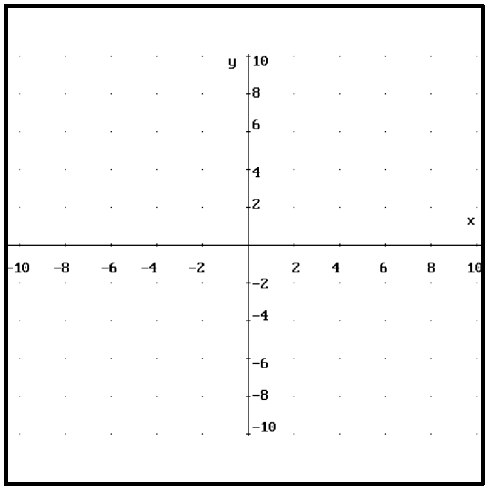
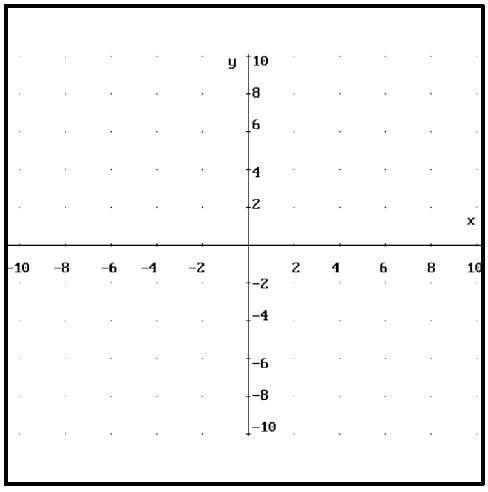


C.



EXTENSION/FOLLOW-UP ACTIVITY

1. You have been working at McArthur Shopping Mall for several months. Your retail job still pays $150.00 plus a 10% commission of your sales per week. In your first three weeks you sold $700 of merchandise, then $300 and $1500 worth of merchandise.

   a. Plot these three salaries as points on a linear graph.

   b. Determine the linear equation that models how your pay is calculated. Graph/sketch this equation.

   c. How are the graphs from the two steps above related? Discuss.

2. Your supervisor comes to you and offers you a new pay scale where you earn $200 plus 7% commission of your sales per week. Should you accept the offer? Why? Why not?